DE2 Electronics 2 for Design Engineers

Tutorial Sheet 1 – Signals in Time and Frequency Domains

SOLUTIONS

1.* Sketch each of the following continuous-time signals. For each case, specify if the signal is causal/non-causal, periodic/non-periodic, odd/even. If the signal is periodic specify its period.

(i) $x(t) = 2\sin(2\pi t)$

(ii) $x(t) = \begin{cases} 3e^{-2t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

(iii) $x(t) = 1/|t|$

SOLUTION:

(i) Non-causal, because it takes non-zero values for $-\infty < t < \infty$. Periodic with period 1. Odd because $x(-t) = -x(t)$.

(ii) Causal, because it takes non-zero values for $0 \leq t < \infty$. Non-periodic. Neither odd nor even.

(iii) Non-causal, because it takes non-zero values for $-\infty < t < \infty$. Non-periodic. Even because $x(-t) = x(t)$.

2.* Sketch the signal

$$x(t) = \begin{cases} 1-t, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Now sketch each of the following and describe briefly in words how each of the signals can be derived from the original signal $x(t)$.

(i) $x(t+3)$

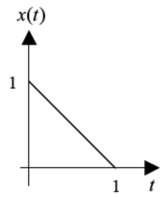
(ii) $x(t/3)$

(iii) $x(t/3+1)$

(iv) $x(-t+2)$

(v) $x(-2t+1)$

SOLUTION:



(i) Left shift by 3.

(ii) Linearly expand by factor of 3.

(iii) $x(t/3+1) = x[(t+3)/3]$. Linearly stress (expand) by factor of 3 and shift left by 3.

(iv) Time reverse and shift right by 2.

(v) $x(-2t+1) = x[-2(t-1/2)]$. Time reverse, linearly compress by factor of 2 and shift right by ½.

3.** Sketch each of the following signals. For each case, specify if the signal is causal/non-causal, periodic/non-periodic, odd/even. If the signal is periodic specify its period.

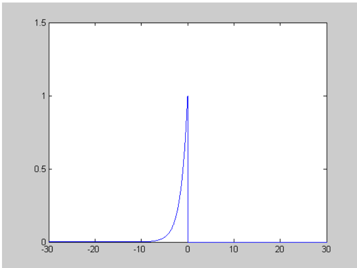
(i) $x[n] = \cos(n\pi)$

(ii) $x[n] = \begin{cases} 0.5^{-n}, & n \le 0 \\ 0, & n > 0 \end{cases}$

SOLUTION:

(i) Non-causal, because it takes non-zero values for $-\infty < n < \infty$. Periodic with period 2. Even because $x[-n] = x[n]$. We all know how it looks like.

(ii) Non-causal, because it takes non-zero values for $-\infty < n \le 0$. Non-periodic. Neither odd nor even.



4.* Sketch the spectrum of the time domain signal.

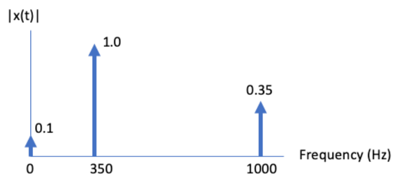
(i) $x(t) = \sin(2\pi \times 350t) + 0.35 \times \sin(6283t) + 0.1$

(ii) $y(t) = 1.5 \times \cos(2199t) + \sin(2\pi \times 1000t \div \pi / 2)$

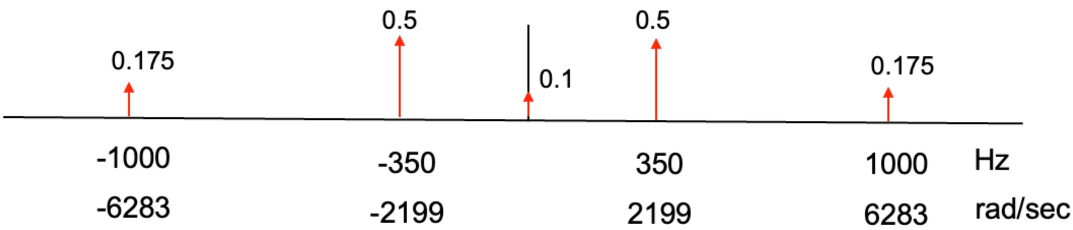
SOLUTION:

(i) We will only consider the magnitude spectrum.

x(t) has three components: dc, and f1 = 350Hz, f2 = 6283/2π = 1000Hz. If we plot the spectrum using only positive frequency axis as amplitude of sine waves, then

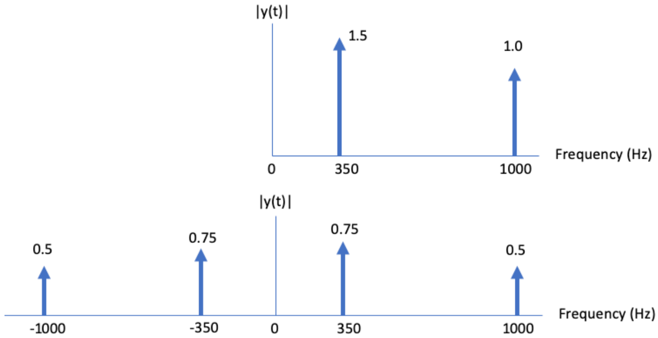


or if we apply Euler's formula and express the spectrum as exponential functions, then the spectrum will be:



(ii) Signal y(t) has the same two frequency components as x(t) with different amplitude. No dc. Hence the two-sided spectrum is:

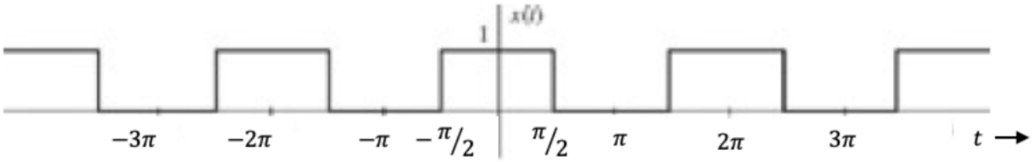
(iii)



5.**

Proof that the Fourier series of the pulse signal shown below is:

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t \ + \ .....\right)$$



Fourier series equation is:

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\,)$$

$T_0 \ = 2\pi$ and $\omega_0 \ = 1$

$$a_0 \ = \ \frac{1}{T_o}\int_{T_o} x(t)dt \ = \ \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} x(t)dt = ½$$

$$a_n \ = \ \frac{2}{T_0}\int_{-\pi/2}^{\pi/2} x(t)\cos n\omega_0 t \, dt \ = \ \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos nt \, dt \ = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

For even, $a_n \ = \ 0$

For n = 1, 5, 9, 13, …. $a_n \ = \ \frac{2}{n\pi}$

For n = 3, 7, 11, 15, ... $a_n \ = \ -\frac{2}{n\pi}$

$$b_n \ = \frac{2}{T_0}\int_{-\pi/2}^{\pi/2} x(t)\sin n\omega_0 t \, dt = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \sin nt \, dt \ = 0$$

Hence,

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t \ + \ .....\right)$$